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ABSTRACT

In the 1970s W. Doise, G. Mugny and A.-N. Perret-Clermont underlined for the first time the essential role played by social interactions in cognitive development. Since then, many authors have been studying social interactions and their mediating role in knowledge apprehension and in skills acquisition. Inspired by L. Vygotsky's theory, many contextualized researches were conducted that showed social interactions, namely peer interactions, were a main facilitator factor for pupils' socio-cognitive development, both in performing their math tasks and in relation to their overall academic achievement in this subject. The contextualized studies also underlined the power of peer interactions in promoting pupils' social integration and participation. Interaction and Knowledge is a research-action project that was implemented in several math classes (5th to 11th grade) with the goal of promoting peer interactions in math classes as a way of changing the didactic contract and facilitating pupils' socialization and school achievement. A deep analysis of peer interactions shows how important the social and cultural aspects of learning mathematics are in pupils' performances in math class. The examples discussed underline the role of many psycho-social factors such as the situation, the social and academic status of the peer, the work instructions that are given, and the didactic contract. The data stress the importance of this kind of analysis if the goal is to promote more positive attitudes toward math and better school performance in this area. (Author/YDS)

Social Interactions and Mathematics Learning

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SOCIAL INTERACTIONS AND MATHEMATICS LEARNING

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Universidade de Lisboa**Abstract**

In the 70s Doise, Mugny and Perret-Clermont (1975, 1976) underlined for the first time the essential role played by social interactions in cognitive development. Since then many authors have been studying social interactions and their mediating role in knowledge apprehension and in skills acquisition. Inspired by Vygotsky's theory, many contextualized researches were conducted and they showed that social interactions, namely peer interactions, were a main facilitator factor for pupils' socio-cognitive development, their performances in Maths tasks and their school achievement in this subject. Contextualized studies also underlined the power of peer interactions in promoting pupils' social integration and participation. Interaction and Knowledge is a research-action project that is implemented in several Maths classes (5th to 11th grade) and whose main goal is to promote peer interactions in the Maths class as a way of changing the didactic contract and facilitating pupils' socialization and school achievement. A deep analysis of peer interactions shows how important the social and cultural aspects of learning mathematics are in pupils' performances in the Maths class. The examples that we are going to discuss underline the role of many psycho-social factors such as the situation, the social and school status of the peer, the work instructions that are given and the didactic contract. Our data stress the importance of this kind of analysis if we want to promote more positive attitudes towards Maths, as well as better school performances.

Theoretical background

Social interactions play a fundamental role in knowledge apprehension and in skills acquisition as well as in socio-cognitive development. The first studies by Doise, Mugny and Perret-Clermont (1975, 1976), still using a *quasi-experimental* design and based on piagetian tasks, clearly underlined the power of social interactions. In these studies children showed more progress when they were interacting while solving the tasks than when they did them individually. But even stronger than that, the promotion of their cognitive development remained stable in time, as they maintained their performances when asked a long time later and when working individually once again.

Vygotsky's (1962, 1978) theory shed light on the essential role played by social

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'interactions, namely when we think about scientific knowledge. The importance of contextualized researches became apparent and school classes have been a privileged stage for research during these two last decades. Tasks were no longer piagetian, but were directly related to curricula contents and psycho-social factors such as the situation, the task, work instructions, the actors involved in the situation, the contents, all of which were deeply analysed. Performances were no longer seen as independent of these psycho-social factors and so social interactions played a significant role in the way they mediated pupils' relations with school knowledge as we may see in a recent literature review (Liverta-Sempio and Marchetti, 1997).

Peer interactions were often studied and they seemed to be quite effective. In the two last decades many studies have stressed their positive effect on pupils' performances and in their school achievement, namely in Mathematics (Perret-Clermont and Nicolet, 1988; Perret-Clermont and Schubauer-Leoni, 1988; Schubauer-Leoni and Perret-Clermont, 1985; Sternberg and Wagner, 1994). Peer interactions were often associated to the socio-cognitive conflict and were seen as a way of implementing a co-construction of knowledge. They seemed to be a powerful way of confronting pupils with one another's solving strategies and that made them decentralize from their own position and discuss the one another's conjectures and arguments.

Knowledge was then conceived as a social construction. Mathematical knowledge was seen as exterior to and pre-existing in the subject and so one of the pupil's tasks was to find out meanings of that knowledge in order to apprehend it. Facing the social dimension of mathematical learning obliged us to conceptualize learning as a much more complex process, in which teachers and pupils played dynamic roles. The relations established among them, the intersubjectivity they were (or were not) able to build (Wertsch, 1991) were main points in pupils' performances and school achievement. Peer interactions promoted better relations among pupils, an increase in their self-esteem and in their ability to construct a common intersubjectivity. Thus, implementing peer interactions within the Maths classes proved to be an effective way of promoting pupils' performances and school achievement (César, 1997; César e Torres, 1997).

The importance of the situations was illustrated by several studies that compared school performances with daily life performances in tasks that were equivalent in their degree of complexity and in the contents they were related to (Carragher, Carragher and Schliemann, 1989; Saxe, 1989; Wistedt, 1994). In all these studies subjects had much better performances in their daily life activities than in school tasks, probably because daily life activities were meaningful to them. But performances may also be different when we change only the work instructions even without changing the situation nor the task, which is extremely important in pedagogical terms (César, 1994, 1995; Nunes, Light and Mason, 1993) and when the didactic contract changes (César, 1997; César and Torres, 1997; Schubauer-Leoni and Perret-Clermont, 1985). It is the didactic contract that legitimates

what both pupils and teachers expect from each other and so it plays a most important role in the way pupils behave, in their self-esteem, in their persistence when they are solving a task, in their performances and in their school achievement.

Vygotsky (1962, 1978) introduced the notion of zone of proximal development (ZPD) and argued that teaching would be much more effective if teachers were able to work with their pupils in that zone. Vygotsky believed that social interactions were powerful, but he thought they were only an efficient way of promoting students' learning and socio-cognitive development if students were interacting with a more competent peer. Recent studies showed that peer interactions are much more powerful in themselves than what Vygotsky conceived, as both in asymmetric and symmetric dyads pupils are able to progress and, more important still, in asymmetric dyads they both progress. This means that there is no need for a more competent peer in order to facilitate better performances, interaction itself is enough.

All these findings can be very important in teachers' practices as it becomes clear that pupils' performances and school achievement are very complex processes. Discovering that the more competent peer could also progress was an essential step to believe that implementing peer interactions in the Maths class during a whole school year or several school years could be a good way of dealing with the degree of underachievement we have in this subject. Pupils' attitudes towards Maths are often very negative, they usually don't believe they can learn this subject and be successful. Thus, implementing a new didactic contract and innovating practices was fundamental if we wanted to promote school achievement in Maths.

To understand the role played by peer interactions in the promotion of a positive self-esteem, more positive attitudes towards Maths, better performances and school achievement we need to carry out a deep analysis of the interactions that take place within the Maths class. Some authors had already done this kind of work but only from a didactic perspective (Brun and Conne, 1990). We decided to do this work in an interdisciplinary team as we wanted to gather different ways of looking at the same reality. This analysis provides information about the mechanisms involved in pupils' communication and performances, about how they build their conjectures and argumentations, about the role played by self-esteem and emotions in all that work, about how they are able to regulate different ideas and solving strategies related to a given problem, about how they negotiate within the dyad and the way they play (or don't play) a leadership role (César, 1997; César e Torres, 1997).

We all live in a social world. Social interactions play a most significant role in our lives. Why should this be forgotten when we are in a school class?

Analysis of a peer interaction

CASE 1 - And if I don't really believe you, can I still learn with you?

Problem - A grocer sold half a cheese, then a quarter and finally a sixth. He then checked that 125gr. were left over. How many kilos did the cheese weigh in the first place?

[V. starts drawing a circumference and then stops to read the problem again.]

M. - What's that?

V. - It's a cheese.

M. - A cheese?... What for?

V. - Now I'm going to draw what he sold...

M. - But I think you do this with sums...

V. I don't know how to do it with sums... so I'm going to see if it works this way...

M. - Then do yours, I'll do mine and then we'll explain.

[Each one uses his own solving strategy. V. uses a graphic representation strategy connected to an arithmetic strategy; M. uses an arithmetic strategy from the start. V. finishes first.]

M. - How's yours?

V. - I drew the cheese, then I divided it into 6 equal parts... so as to be easier... Get it?

M. - More or less... I understand what you did, but I haven't yet figured out why you divided the cheese in 6 parts and not in 2... first he sells half...

V. - I know... but I had to know how to mark a half, a quarter and a sixth... a half and a quarter is easy... the hardest one is the sixth, so I started with that... or else I wouldn't know how to go about it, after marking the half and the quarter I wouldn't see where a sixth was...

M. - Sorry?

V. - Draw a circle! [M. draws the circle]

V. - Now, mark half, which is what you sold. [M. does as he tells her to]

V. - Now mark another quarter, which is the other bit you sold. [She does]

V. - Now mark a sixth, which is the third bit you sold. [M. stops, pencil in hand and says]

M. - Oh! I see! It's much harder like this... do yours! It must be better.

[V. draws the cheese again, divides it in six parts, traces half, then a quarter and a sixth. M. follows his steps carefully and says the numbers out loud as he goes along. Then he says]

V. - See what's not traced?

M. - Yes.

V. - I think it's half of a sixth... so it's $1/12$. If $1/12$ is 250gr, then the whole cheese is $250\text{gr} \times 12$, which is 1500 gr [He had done the sum on the calculator]. That's 1,5 kg.

M. - But I didn't get that!

V. - How did you do it?

M. - With sums. I added $1/2 + 1/4 + 1/6$ and ended up with $3/12$. That's what he sold. One cheese is $12/12$. So I subtracted $3/12$ from $12/12$ and got $9/12$, which are 250gr... but now I don't know how to go on.

V. - I don't understand your sums because I don't know maths... but you've got that wrong... because you say he sold $3/12$ and that's a quarter of the cheese...

M. - Don't be dumb! No it's not... It's the sum of all that...

V. - You wish, but that's not what your sum did... See... [He draws another cheese, divides it into 12 parts and marks 3. Then he looks at M.]

M. - What a mess! I can't understand why... the sums should also give...

Teacher - How are we doing here?

M. - He says he's right, and when he does the drawing he seems to be, but I think this is maths, we should do it with sums!... [V. explains what he did]

Teach. - Do you understand how he did that?

M. - Yes.

Teach. - How about you? Do you understand what she did?

V. - I just understand that her sums are wrong... I drew it here and it's only $1/4$... but I think it should also be possible that way... but I don't know any of this, I don't know how to do it that way...

Teach. - So, M., how did you think this out?

M. - That I had to add everything he sold to see how much I got... $1/2 + 1/4 + 1/6$

V. - So far I agree.

M. - And I got $3/12$...

Teach. - $3/12$?

M. - Yes... $1 + 1 + 1 = 3$ and $2 + 4 + 6 = 12$

Teach. - And how do you add fractions? [Silence]

Teach. - What do you need to do to be able to add fractions?

M. [Hesitating] - Reduce them to the same number here? [She points to the denominators]

Teach. - Of course!

M. - Oh, then I know!... It's 6... No, it's 12. $6/12 + 3/12 + 2/12 = 11/12$

V. - Right... so you were left with $1/12$ after all, like me! [He's visibly happy]

M. - Yes... then you just have to do the same sums you did.

V. - After all I was the one who was right this time! [Victorious] I think I'd never got anything right in maths before... by myself.

The students who established this interaction were in the 9th grade. M. was considered a

good student in Mathematics, while V. had repeated failures in this subject (mark 1, which is the lowest possible), thinking it wasn't even worth trying, because he knew nothing and was incapable of learning, as he explained at the start of the year. This interaction takes place about two weeks after classes began and this pair had been formed because M. normally used an analytical reasoning, grasped previous years' contents better, but had difficulty whenever problems implied geometrical reasoning or called for a good mathematical intuition. Besides, she was convinced she was one of the best students of her class, and that she was damaged by some of the students' slow rhythm and by others' lack of interest. V. had an extremely low self-esteem in the beginning of the year, but had reacted positively to the few successes he had already had in the short period of classes until then. He had demonstrated that, if stimulated, he could have great ideas about problem solving, his mathematical intuition was very good, and found it easy to visualise situations that required this, but lacked a great deal of knowledge content-wise. Our expectation was that both, quite suspicious of each other at the start, would discover what interaction with one another could offer them in terms of personal progress.

When the problem is set forth, one of the features we had identified in V. becomes apparent straight away: he does not know how to solve the problem through calculations, so he uses a graphic representation - *he draws the cheese*. Naturally at this stage of the school year, M. feels very sure that she knows the right path towards the correct solution. "*A cheese?... What for? (...) But I think you do this with sums... (...) Then do yours, I'll do mine and then we'll explain*". She is the leader, in the sense that she is who decides that what V. is doing is no good and it is also she who decides that it's best that they work separately and only interact afterwards. At this point his lack of faith in his ability to solve the problem is still overwhelming. But we can see that V. is starting to gain some confidence in his abilities and is knowing how to deal better with the limitations that come from not knowing a lot of the contents. "*It's a cheese... (...) Now I'm going to draw what he sold... (...) I don't know how to do it with sums... so I'm going to see if it works this way...*". A week ago, V. would have erased everything and stayed still, doing nothing, as soon as he heard M. say: "*A cheese?... What for?...*". Now, he already knew it was worth it to keep trying.

So, this brief moment of initial interaction, when they decide how they are going to work, is followed by a moment with no interaction, during which each one follows the solving strategy he/she picked. However, as soon as she has finished, M. turns to V. and asks him how he did it. We are not sure, but we believe she was impressed by the fact that he finished first and by the happy expression on his face. But she also knew the work instructions set by the teacher, so she might just be doing her part as the obedient student she was.

V. begins his explanation, concerned that M. understands all that he did, each decision he made. However, their understanding is not always easy: she wanted him to follow the elements in the problem step by step: "*(...) I understand what you did, but I haven't yet*

figured out why you divided the cheese in 6 parts and not in 2... first he sells half..."; as V. could visualise easily, he quickly understood that the difficulty lay in tracing $1/6$, not one half: "(...) a half and quarter is easy... the hardest one is the sixth, so I started with that... (...)". In this manner, he had decided to begin his graphic representation according to what he had understood... which was not at all obvious to M., hence her exclamation: "Sorry?"

Since M. is not managing to follow his reasoning, V. changes his strategy and decides to tell her to do as she pleases, step by step, so as to be confronted with the final difficulty. At this moment, V. clearly takes charge of the process: he is the only one giving instructions and M. obeys. The strategy chosen by V. works in full for, as she arrives at $1/6$, M. becomes still, holding her pencil in the air, not knowing how to continue. This has the effect V. intended: she understands he had some reason in what he did and decides to listen to him carefully, instead of trying to convince him that only she knows best. She even praises him for the first time: "(...) Do yours! It must be better." Here we find an interesting interactive process: the most competent element of the two, in terms of previous years' contents, loses the dyad's leadership; and it is V.'s enormous mathematical intuition and his ability to visualise which lead him to an increasingly important role during interaction with his pair.

The way V. continues his explanation is thrilling, especially for a student who says he *"doesn't know maths"*. He looks at the diagram he drew and says *"I think it's half of a sixth... so it's $1/12$. If $1/12$ is 250gr, then the whole cheese is $250\text{gr} \times 12$, which is 1500gr. That's 1.5Kg."* V.'s ability to visualise is, indeed, extraordinary. He does not know how to work with fractions but, when he looks, he can see that what is left is half of $1/6$ and, just by looking at the cheese he drew, he can immediately see that that would correspond to having divided it into 12 parts and taking one of them.

It is amazing to see that a student with these abilities has always failed at Mathematics since the 5th grade and that last year's teacher described him as *"incapable of mathematical reasoning and totally ignorant"*. In fact, at the beginning of the year, V. was convinced of this himself... but he quickly began to change his opinion.

The same does not apply to M., who seemed willing to listen to him and collaborate with him, as long as her wisdom was not questioned. Therefore, when she saw that V.'s result was different from hers, she hurriedly exclaimed *"But I didn't get that!"*, despite not having been able to even finish solving the problem.

V. asks her what she did and she answers *"I added $1/2 + 1/4 + 1/6$ and ended up with $3/12$. That's what he sold."* V.'s answer is quite revealing: *"I don't understand your sums because I don't know maths... but you've got that wrong... because you say he sold $3/12$ and that's a quarter of the cheese..."*. That is, he presumes - and continued to be completely certain of this, at that time of the school year - that he does not know

Mathematics, but he has made considerable progress: he no longer believes that he does not know how to think. Therefore, he does not know how to correct M.'s sums, but he is sure they are wrong. Through the graphic representation he draws, he knows very well that $3/12$ are the same as $1/4$, so her sums cannot be right.

But M. does not readily admit that she is wrong. After all, she is a good student, the one who usually knows the answers, and is not willing to let a couple of cheese sketches defy her wisdom. So she hastens to reply: *"Don't be dumb! No it's not... It's the sum of all that..."*. And V., who does not want to get mad at her and probably knows all too well how frustrating it is to make a mistake when you think you are right, answers back without arguing, but with extreme subtlety: *"You wish, but that's not what your sum did... See..."*. And he goes back to his graphic representations to prove to M. that what he is saying is correct. Faced with this proof, M. becomes really confused and all she can say is she does not understand what happened and that her sums should also do.

At this crucial moment the teacher, who has been going around the room looking at what each pair has done and is not aware of what is happening between V. and M., arrives. It would have been interesting to have seen what they would have done if the teacher had not turned up then, how they would overcome this dilemma. But a contextualised investigation is just that: it happens on a stage which is the classroom, in a dynamic social climate that does not always develop in a manner that allows us to observe all that we wish, how we wish.

It is important to note that, as soon as the speaker is the teacher - the most competent person in the classroom, the one who gave the work instructions and who evaluates, which is still a big concern for M. - it is M. who talks to him, trying to make him back her idea that *"This is maths, we should do it with sums!..."*. It is worthy of notice that the teacher does not support M.'s claim, but does not criticise it either. He is more concerned about finding out if each one managed to understand the strategy used by the other. Only when the teacher speaks directly to him does V. reply, taking part in the dialogue, which is now between the three of them. V. reveals a great humbleness: *"but I don't know any of this, I don't know how to do it that way..."* because he does not know how to work with fractions. However, he does know that his *"drawings"* are right and that M.'s *"sums"* are wrong. But since V. has no past history of success in Mathematics, he is perfectly willing to accept that there are alternative strategies. On top of this, he does not know Mathematics, as he often states, but he is already capable of finding certain strategies to solve the tasks proposed. For him there is no question that Mathematics can be done through sums, but he thinks - and rightly so - that in that case the result should be the same.

Once again, the teacher avoids any sort of judgement and asks M. how she thought things out again. She explains that: *" $1/2 + 1/4 + 1/6$ (...) and I got $3/12$ (...) Yes... $1 + 1 + 1 = 3$ and $2 + 4 + 6 = 12$ "*. From this the teacher asks how fractions are added. Since he receives

no answer from either of them, the teacher decides to reword the question, which works, for M. can remember that it is necessary to reduce fractions to a common denominator, although her language is not very rigorous. At this point of interaction, the teacher has called for knowledge from previous years and, as might be expected in this dyad, M. was the one who answered. But it is important to highlight that V. kept on listening with all his attention.

As soon as M. does her sums and reaches the result of $11/12$ for all that has been sold, V. looks visibly happy and exclaims: *"Right... so you were left with $1/12$ after all, like me!"* To which M. adds: *"Yes... then you just have to do the same sums you did."* And finally, with a victorious look on his face, he said *"After all I was the one who was right this time! ?Victorious? I think I'd never got anything right in maths before... by myself."*

After this episode, V. began to participate more and more. He did not just copy from the blackboard during the stage of class debate, or when the teacher explained something. He would ask for more explanations until he had understood. For a while, he would keep apologising and stating that *he "knew nothing about Maths... ?he? just wanted to understand"*. Sometimes he would say *"When I see things, I can do them"*. But at the same time he revealed a great ability to grasp knowledge he did not have. In this case, he managed to learn how to add fractions. And he never forgot again that it was necessary to reduce them to a common denominator. In order to learn how to divide, we set him a challenge: he would go home and think that if, as he put it, *" $1/6 \div 2 = 1/12$ "*, then what was the rule for dividing fractions? To our amazement, V. did not go home to think. He stayed there, during breaktime, and asked with a suspect look whether *"you could swap downwards and upwards"* and, as I did not reply but simply smiled, he said: *"I don't get any of this... but maybe it's like this... the first one stays as it is... this one ?pointing to the second fraction? rolls upside down... and then what's really weird... because it seems that instead of dividing you multiply... nah... can't be... but I don't see any other way"*. And he also never again forgot that in order to divide fractions he had to turn the second one upside down, and was more and more excited by the fact that Mathematics could be learnt by seeing. After all, learning Mathematics was much more fun and much easier than he had ever imagined!

Concluding remarks

To promote peer interactions in the Maths class it is not enough to sit pupils side by side - we also need to define the criteria for choosing the peers. When we put M. and V. in a dyad we were expecting them to be suspicious of each other, but we also knew they had different kinds of solving strategies and abilities and we hoped they would discover how

- useful their interaction could be for their performances and their school achievement. Anyhow this would only function if we were able to implement a new didactic contract within the class and if they accepted it.

Although suspicious - each one follows his/her own solving strategy in the beginning - they followed the rules implemented by their teacher: they had to be able to explain both solving strategies and any of them could be the one who was going to present the dyad work during the general discussion. So, as soon as they finished each one's solving strategy, they were interested in explaining and understanding what their peer had been doing alone. And they don't merely hear what the other one is saying, they are really listening carefully and they ask questions and argue each time they don't agree or don't understand the reasons why their peer solved the problem that way.

This didactic contract enables them to show each other their abilities and their difficulties. It makes each of them take the role of leader in different parts of the interaction and so M. is confronted with the fact that sometimes she fails too and that V. may have good ideas, while V. becomes more and more confident about his abilities. Peer interaction is a fine way of stimulating pupils' autonomy, which is quite visible in this case. Pupils work by themselves for a long time and they are able to regulate their ways of solving the task. Even when the teacher approaches them they know he wouldn't give them answers, he will mostly ask them questions and they are ready to answer him.

This is just one case chosen among many others, but after four years work in 26 different classes, from the 5th to the 11th grades, we are convinced that peer interactions are an effective way of promoting pupils' positive attitudes towards Maths, their self-esteem and socio-cognitive development, better affective relations in the class and their achievement in Maths. It is also a way of exploring each pupil's abilities and of making their past underachievement and their socio-cultural differences less determinant. Pupils accept each other more easily and they can profit from their differences instead of being penalized by them. Their representations about Maths change and they become more deeply engaged in their school activities. But, above all, for the first time many of them are able to have a future life project in which school has a role to play.

Note

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